Critical behavior of certain antiferromagnets with complicated ordering: Four-loop ε-expansion analysis

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Abstract

The critical behavior of a complex N-component order parameter Ginzburg-Landau model with isotropic and cubic interactions describing antiferromagnetic and structural phase transitions in certain crystals with complicated ordering is studied in the framework of the four-loop renormalization group (RG) approach in $(4-\varepsilon)$ dimensions. By using dimensional regularization and the minimal subtraction scheme, the perturbative expansions for RG functions are deduced and resummed by the Borel-Leroy transformation combined with a conformal mapping. Investigation of the global structure of RG flows for the physically significant cases $N = 2$ and $N = 3$ shows that the model has an anisotropic stable fixed point governing the continuous phase transitions with new critical exponents. This is supported by the estimate of the critical dimensionality $N_c = 1.445(20)$ obtained from six loops via the exact relation $N_c = \frac{1}{2}n_c$ established for the complex and real hypercubic models.

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I Introduction

In this paper we investigate the critical properties of phase transitions in certain antiferromagnets involving an increase of the unit cell in one or more directions at the critical temperature. They are known to be described by a generalized $2n$-component ($n \geq 2$) Ginzburg-Landau model with three independent quartic terms

$$\begin{aligned} H &= \int d\,^D x \Big[\frac{1}{2}\sum_{i=1}^{2n}(m_0^2\,\varphi_i^2 + \vec{\nabla}\varphi_i\vec{\nabla}\varphi_i) + \frac{u_0}{4!}\Big(\sum_{i=1}^{2n}\varphi_i^2\Big)^2 \\ &+ \frac{v_0}{4!}\sum_{i=1}^{2n}\varphi_i^4 + \frac{2z_0}{4!}\sum_{i=1}^{n}\varphi_{2i-1}^2\varphi_{2i}^2\Big] \end{aligned} \quad (1)$$

associated with isotropic, cubic, and tetragonal interactions, respectively [1]. Here φ_i is the real vector order parameter in $D = 4 - \varepsilon$ dimensions and m_0^2 is proportional to the deviation from the mean-field transition point. According to the universality hypothesis, the critical phenomena in various physical systems must not depend on microscopic details, but are determined solely by the spatial dimensionality of the system, the interaction range as well as the symmetry and the dimensionality of the order parameter field. The anomalous behavior of the thermodynamic quantities and the correlation radius r_c are expressed by power laws in the critical region.

When $n = 2$, Hamiltonian (1) describes the antiferromagnetic phase transitions in $TbAu_2$ and DyC_2 and the structural phase transition in the NbO_2 crystal. The phase transitions in the helical magnets Tb, Dy, and Ho belong to the same class of universality [2]. Another physically important case $n = 3$ is relevant to the antiferromagnetic phase transitions in such substances as K_2IrCl_6, TbD_2, and Nd. The phase transition in antiferromagnet MnS_2 belongs to the same class of universality [2]. All these phase transitions are predicted to be of second order[3] [2, 5]. This is confirmed by several experiments (see Ref. [6] and references therein). However the experimental data were insufficiently accurate to provide reliable values of critical exponents and the obtained estimates [7, 8, 9] were found to differ significantly from the theoretically expected numbers. Such a distinction should be interpreted as a manifestation of influence of various possible effects in the real substances such as the presence of defects.

For the first time, the magnetic and structural phase transitions described by model (1) were studied in the framework of the renormalization group (RG) by Mukamel and Krinsky within the lowest orders in ε [1]. A three-dimensionally stable fixed point (FP) with coordinates $u^* > 0$, $v^* = z^* > 0$ was predicted[4]. That point was shown to determine a new universality class with a specific set of critical exponents. However, for the physically important case $n = 2$, the critical exponents of the unique point turned out to be exactly the same as those of the $O(4)$-symmetric one.

[3] An interesting type of multisublattice antiferromagnets, such as MnO, CoO, FeO, and NiO, was studied in Ref. [3, 4, 5]. It was shown, in the leading orders in ε, that the phase transitions in these substances are of first order.

[4] Following Mukamel [1], we call this point "unique".

For the years, an alternative analysis of critical behavior of the model, the RG approach in three dimensions, was carried out within the two- and three-loop approximations [10, 11]. Those investigations gave the same qualitative predictions: the unique stable FP does exist on the 3D RG flow diagram. By using different resummation procedures, the critical exponents computed at this point proved to be close to the exponents of the Bose FP ($u = 0$, $v = z > 0$) rather than the isotropic ($O(n)$-symmetric; $u > 0$, $v = z = 0$) one. It was also shown that the unique and Bose FPs are very close to each other, so that they may interchange their stability in the next orders of RG approximation [11].

Recently, the critical properties of the model were analyzed in third order in ε [12, 13]. Investigation of the FP stability and calculation of the critical dimensionality n_c of the order parameter, separating two different regimes of critical behavior[5], confirmed that model (1) has the unique stable FP at $n = 2$ and $n = 3$. However, the twofold degeneracy of the stability matrix eigenvalues at the one-loop level was observed for this FP [13]. That degeneracy was shown to cause a substantial decrease of the accuracy expected within the three-loop approximation and powers of $\sqrt{\varepsilon}$ to appear in the expansions[6]. So, computational difficulties were shown to grow faster than the amount of essential information one may extract from high-loop approximations. That resulted in the conclusion that the ε-expansion method is not quite effective for the given model.

Another problem associated with model (1) is the question whether the unique FP is really stable in 3D, thus leading to a new class of universal behavior, or its stability is only an effect of insufficient accuracy of the RG approximations used. Indeed, there are general nonperturbative theoretical arguments indicating that the only stable FP in 3D may be the Bose one and the phase transitions of interest should be governed by that stable FP [15]. The point is that when $v = z$, the model (1) describes n interacting Bose systems. As was shown by Sak [16], the interaction term can be represented as the product of the energy operators of various two-component subsystems. It was also found that one (the smallest) of the eigenvalue exponents characterizing the evolution of this term under the renormalization group in a neighborhood of the Bose FP is proportional to the specific heat exponent α. Since α is believed to be negative at that point, and that is supported experimentally [17] and theoretically [18, 19, 20, 21], the interaction is irrelevant. Consequently, the Bose FP should be stable in three dimensions. However, up to now, this conclusion found no confirmation within the RG approach, so the model (1) poses a certain challenge to the perturbative methods in the theory of critical phenomena. It is therefore highly desirable to extend already known ε expansions for the stability matrix eigenvalues, critical exponents and the critical dimensionality in order to apply more sophisticated resummation techniques to longer expansions.

Thus, in the present work we firstly avoid the problem of the eigenvalues degeneracy in model (1) by analyzing the critical behavior of the equivalent complex N-component order

[5] When $n > n_c$ the unique FP is stable in 3D while for $n < n_c$ the stable FP is the isotropic one.

[6] Similar phenomenon was observed earlier in studying the impure Ising model (see Ref. [14]). Half-integer powers in ε arising in that model have different origin but also lead to the loss of accuracy.

parameter model with the effective Hamiltonian

$$H = \int d^D x \Big[\frac{1}{2}\sum_{i=1}^{N}(m_0^2\psi_i\psi_i^* + \vec{\nabla}\psi_i\vec{\nabla}\psi_i^*) + \frac{u_0}{4!}\sum_{i,j=1}^{N}\psi_i\psi_i^*\psi_j\psi_j^* + \frac{v_0}{4!}\sum_{i=1}^{N}\psi_i\psi_i\psi_i^*\psi_i^*\Big] \qquad (2)$$

comprising the isotropic and cubic interactions[7]. Note that this Hamiltonian also determines the real hypercubic model [23] relevant to the magnetic and structural phase transitions in a cubic crystal if ψ_i is thought to be the real n-component order parameter. The model (2) comes out exactly from model (1) at $v_0 = z_0$ and it is free from the eigenvalues degeneracy. Secondly, we examine the existence of an anisotropic stable FP in model (2) on the basis of the higher-order ε expansion. Namely, using dimensional regularization and the minimal subtraction scheme [24], we derive the four-loop RG functions as power series in ε and analyze the FP stability. For the first time, we give numerical estimates for the stability matrix eigenvalues from ε expansions using the Borel-Leroy transformation with a conformal mapping [25, 26]. This allows us to carry out the careful analysis of the stability of all the FPs of the model. We establish the exact relation $N_c = \frac{1}{2}n_c$ between the critical (marginal) spin dimensionalities of the complex and the real hypercubic models and obtain the estimate $N_c = 1.445(20)$ using six-loop results of Ref. [27]. We show that the anisotropic (complex cubic; $u \neq 0$, $v \neq 0$) stable FP of model (2), being a counterpart of the unique point in model (1), does exist on 3D RG flow diagram at $N > N_c$. For this stable FP we give more accurate critical exponents estimates in comparison with the previous three-loop results [13] by applying the summation technique of Ref. [28] to the longer series.

II Four-loop ε-expansions, resummation and the fixed point stability

To deduce RG expansions for the β-functions and critical exponents one needs to calculate a set of Feynman graphs, each of them comprising three factors: the combinatorial coefficient, the result of tensor contractions and the integral value associated with the diagram. The combinatorial factors and the values of integrals were found earlier in Ref. [26]. To evaluate the tensor contractions for the vertex and mass diagrams, the tensors G_1 and G_2 corresponding to the isotropic and cubic interactions in Hamiltonian (2) are introduced; they have a simple symmetrized form:

$$\begin{aligned} G_{1\,ijkl}^{\;\alpha\beta\mu\nu} &= \frac{1}{3}\Big(\delta^{\alpha\beta}\delta^{\mu\nu}\delta_{ij}\delta_{kl} + \delta^{\alpha\mu}\delta^{\beta\nu}\delta_{ik}\delta_{jl} + \delta^{\alpha\nu}\delta^{\beta\mu}\delta_{il}\delta_{kj}\Big)\ , \\ G_{2\,ijkl}^{\;\alpha\beta\mu\nu} &= \delta^{\alpha\beta}\delta^{\alpha\mu}\delta^{\alpha\nu}\delta_{ij}\delta_{ik}\delta_{il}\ , \end{aligned}$$

where $\{\alpha, \beta, \mu, \nu\} = 1, 2$ and $\{i, j, k, l\} = 1, \ldots, N$.

[7] The model with the complex vector order parameter was considered by Dzyaloshinskii [22] in studying the phase transitions in DyC_2, $TbAu_2$ ($N = 2$) and TbD_2, MnS_2, and Nd ($N = 3$).

Further, normalizing conditions on the renormalized one-particle irreducible inverse Green functions $\Gamma_R^{(2)}$ and vertices $\Gamma_R^{(4)}$ given by corresponding Feynman diagrams must be imposed. Within the massless theory they are as follows

$$\begin{array}{lcl} \Gamma_R^{(2)}(\{p\};\mu,u,v)\Big|_{p^2=0} & = & 0 \quad , \\ \frac{\partial}{\partial p^2}\Gamma_R^{(2)}(\{p\};\mu,u,v)\Big|_{p^2=\mu^2} & = & 1 \quad , \\ \Gamma_{UR}^{(4)}(\{p\};\mu,u,v) & = & \mu^\varepsilon u \quad , \\ \Gamma_{VR}^{(4)}(\{p\};\mu,u,v) & = & \mu^\varepsilon v \end{array} \tag{3}$$

with one more condition on the $|\psi|^2$ insertion

$$\Gamma_R^{(1,2)}(\{p\},\{q\};\mu,u,v)\Big|_{\substack{p^2=q^2=\mu^2 \\ pq=-\frac{1}{3}\mu^2}} = 1 \ . \tag{4}$$

Here m, u, and v are the renormalized mass and dimensionless coupling constants, μ is an arbitrary mass parameter introduced for dimensional regularization. The vertices $\Gamma_u^{(4)}$ and $\Gamma_v^{(4)}$ are connected with the vertex function without external lines

$$\Gamma^{(4)}{}_{ijkl}^{\ \alpha\beta\mu\nu} = \Gamma_u^{(4)} \cdot G_{1\,ijkl}^{\ \alpha\beta\mu\nu} + \Gamma_v^{(4)} \cdot G_{2\,ijkl}^{\ \alpha\beta\mu\nu} \ .$$

¿From renormalization conditions (3) and (4), the expansions for the renormalization constants Z_ψ, Z_u, Z_v, and $Z_{|\psi|^2}$ may be obtained. These constants relate the bare mass m_0 and coupling constants u_0, v_0 of the Hamiltonian (2) to the corresponding physical parameters:

$$m_0^2 = \frac{Z_{|\psi|^2}}{Z_\psi} m^2 = Z_m m^2 \ , \quad u_0 = \mu^\varepsilon \frac{Z_u}{Z_\psi^2} u \ , \quad v_0 = \mu^\varepsilon \frac{Z_v}{Z_\psi^2} v \ . \tag{5}$$

So, with relations (5) taken into account, the β-functions and critical exponents can be calculated via the formulas

$$\begin{array}{lcl} \frac{\partial \ln u_0}{\partial u}\beta_u + \frac{\partial \ln u_0}{\partial v}\beta_v & = & -\varepsilon \ , \\ \frac{\partial \ln v_0}{\partial u}\beta_u + \frac{\partial \ln v_0}{\partial v}\beta_v & = & -\varepsilon \ , \end{array} \tag{6}$$

$$\begin{array}{lcl} \eta(u,v) & = & 2\frac{\partial \ln Z_\psi}{\partial u}\beta_u + 2\frac{\partial \ln Z_\psi}{\partial v}\beta_v \ , \\ \eta_2(u,v) & = & 2\frac{\partial \ln Z_{|\psi|^2}}{\partial u}\beta_u + 2\frac{\partial \ln Z_{|\psi|^2}}{\partial v}\beta_v \end{array} \tag{7}$$

where $\beta_g \equiv \frac{\partial g}{\partial |\ln \mu|}$, $g = \{u, v\}$. The critical exponents η and η_2 are found by substituting zeros of the β-functions into expressions (7). The critical exponent γ is calculated through the known scaling relation $\gamma^{-1} = 1 + \frac{\eta_2}{2-\eta}$.

The four-loop ε expansions for the β-functions of model (2) were recently obtained in Ref. [29]. They read

$$\beta_u \ = \ \varepsilon u - u^2 - \frac{4}{N+4}uv + \frac{1}{(N+4)^2}\Big[3u^3(3N+7) + 44u^2v + 10uv^2\Big]$$

$$
\begin{aligned}
&- \frac{1}{(N+4)^3}\Big[\frac{u^4}{4}\Big(48\zeta(3)(5N+11)+33N^2+461N+740\Big)\\
&+ u^3v\Big(384\zeta(3)+79N+659\Big)+\frac{u^2v^2}{2}\Big(288\zeta(3)+3N+1078\Big)+141uv^3\Big]\\
&- \frac{1}{(N+4)^4}\Big[\frac{u^5}{12}\Big(-48\zeta(3)(63N^2+382N+583)+144\zeta(4)(5N^2\\
&+ 31N+44)-480\zeta(5)(4N^2+55N+93)+5N^3-3160N^2\\
&- 20114N-24581\Big)-\frac{2u^4v}{3}\Big(12\zeta(3)(3N^2+276N+1214)-36\zeta(4)\\
&\times (19N+85)+\zeta(5)(2400N+23040)-28N^2+3957N+15967\Big)\\
&- \frac{u^3v^2}{3}\Big(72\zeta(3)(19N+426)-4032\zeta(4)+39840\zeta(5)+1302N+46447\Big)\\
&+ \frac{2u^2v^3}{3}\Big(60\zeta(3)(N-84)-792\zeta(4)-4800\zeta(5)-125N-12809\Big)\\
&- \frac{uv^4}{2}\Big(400\zeta(3)+768\zeta(4)+3851\Big)\Big]\ , \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
\beta_v &= \varepsilon v-\frac{1}{N+4}(6uv+5v^2)+\frac{1}{(N+4)^2}\Big[u^2v(5N+41)+80uv^2+30v^3\Big]\\
&- \frac{1}{(N+4)^3}\Big[\frac{u^3v}{2}\Big(96\zeta(3)(N+7)-13N^2+184N+821\Big)\\
&+ \frac{u^2v^2}{4}\Big(4032\zeta(3)+59N+5183\Big)+uv^3\Big(768\zeta(3)+1093\Big)+\frac{v^4}{2}\Big(384\zeta(3)+617\Big)\Big]\\
&- \frac{1}{(N+4)^4}\Big[\frac{u^4v}{4}\Big(48\zeta(3)(N^3-12N^2-140N-567)+144\zeta(4)(2N^2\\
&+ 17N+45)-3360\zeta(5)(3N+13)-29N^3-28N^2-6958N-19679\Big)\\
&+ \frac{u^3v^2}{3}\Big(12\zeta(3)(9N^2-591N-7028)+\zeta(4)(3528N+21240)-480\zeta(5)\\
&\times (10N+287)+61N^2-5173N-66764\Big)-\frac{u^2v^3}{3}\Big(1800\zeta(3)(N+62)\\
&- 144\zeta(4)(8N+203)+172800\zeta(5)+56N+93701\Big)-4uv^4\Big(5090\zeta(3)\\
&- 1296\zeta(4)+7600\zeta(5)+4503\Big)+\frac{v^5}{2}\Big(-8224\zeta(3)+1920\zeta(4)\\
&- 12160\zeta(5)-7975\Big)\Big] \qquad (9)
\end{aligned}
$$

where $\zeta(3)$, $\zeta(4)$, and $\zeta(5)$ are the Riemann ζ functions. For the critical exponents we find

$$
\begin{aligned}
\eta &= \frac{1}{(N+4)^2}\Big[u^2(N+1)+4uv+2v^2\Big]\\
&- \frac{1}{(N+4)^3}\Big[\frac{u^3}{2}(N+1)(N+4)+3u^2v(N+4)+15uv^2+5v^3\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{1}{(N+4)^4}\Big[-\frac{5}{4}u^4(N+1)\Big(N^2-9N-25\Big) - 10u^3v\Big(N^2 \\
&- \quad 9N-25\Big) + 495u^2v^2 + 330uv^3 + \frac{165}{2}v^4\Big] \ , \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\eta_2 \ = \ & -\frac{1}{N+4}\Big[2u(N+1)+4v\Big] + \frac{1}{(N+4)^2}\Big[6u^2(N+1)+24uv+12v^2\Big] \\
& - \ \frac{1}{(N+4)^3}\Big[\frac{u^3}{2}\Big(31N^2+146N+115\Big) + 3u^2v(31N+115) \\
& + \ 3uv^2(N+145) + 146v^3\Big] + \frac{2}{(N+4)^4}\Big[\frac{u^4}{6}\Big(12\zeta(3)(3N^3+8N^2 \\
& + \ 22N+17) + 36\zeta(4)(5N^2+16N+11) - 4N^3 + 977N^2 \\
& + \ 3016N+2035\Big) + \frac{4u^3v}{3}\Big(12\zeta(3)(3N^2+5N+17) + \zeta(4)(180N \\
& + \ 396) - 4N^2 + 981N + 2035\Big) + 4u^2v^2\Big(\zeta(3)(18N+132) \\
& + \ \zeta(4)(18N+270) + 115N + 1391\Big) - \frac{2uv^3}{3}\Big(60\zeta(3)(N-11) \\
& - \ 1152\zeta(4) - 125N - 5899\Big) + 4v^4\Big(25\zeta(3)+48\zeta(4)+251\Big)\Big] \ , \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{-1} \ = \ & 1 - \frac{1}{(N+4)}\Big[u(N+1)+2v\Big] + \frac{3}{(N+4)^2}\Big[u^2(N+1)+4uv+2v^2\Big] \\
& - \ \frac{1}{4(N+4)^3}\Big[3u^3(N+1)(11N+39) + 18u^2v(11N+39) \\
& + \ 10uv^2(N+89) + 300v^3\Big] + \frac{1}{12(N+4)^4}\Big[u^4\Big(24\zeta(3)(N+1)(3N^2 \\
& + \ 5N+17) + 72\zeta(4)(N+1)(5N+11) - 5(N+1)(N^2-399N \\
& - \ 820)\Big) + 8u^3v\Big(24\zeta(3)(3N^2+5N+17) + \zeta(4)(360N+792) \\
& - \ 5(N^2-399N-820)\Big) + 6u^2v^2\Big(48\zeta(3)(3N+22) \\
& + \ 144\zeta(4)(N+15) + 953N + 11227\Big) - 2uv^3\Big(240\zeta(3)(N-11) \\
& - \ 4608\zeta(4) - 515N - 23845\Big) + 12v^4\Big(100\zeta(3)+192\zeta(4)+1015\Big)\Big] \ . \qquad (12)
\end{aligned}
$$

¿From the system of equations $\beta_u(u^*, v^*) = 0$, $\beta_v(u^*, v^*) = 0$, one can calculate all the FPs of model (2), which are given by the power series in ε:

$$
u^* = u^*(\varepsilon) = \sum_{k=1}^{\infty} u_k \varepsilon^k \ , \qquad v^* = v^*(\varepsilon) = \sum_{k=1}^{\infty} v_k \varepsilon^k \ .
$$

There exist four FPs, one of them (Gaussian) is trivial:

1. Gaussian FP

$$u^* = v^* = 0\ .$$

2. Isotropic or $O(N)$-symmetric FP

$$\begin{aligned}
u^* &= \varepsilon + \tfrac{3}{(N+4)^2}(3N+7)\varepsilon^2 - \tfrac{1}{4(N+4)^4}\Big(48\zeta(3)(N+4)(5N+11) \\
&+ 33N^3 - 55N^2 - 440N - 568\Big)\varepsilon^3 \\
&+ \tfrac{1}{12(N+4)^6}\Big(48\zeta(3)(N+4)\Big(63N^3 - 41N^2 - 949N - 1133\Big) \\
&- 144\zeta(4)(N+4)^3(5N+11) + 480\zeta(5)(N+4)^2\Big(4N^2 \\
&+ 55N + 93\Big) - 5N^5 - 1335N^4 - 2(698N^3 - 3299N^2 \\
&- 9666N - 8278\Big)\varepsilon^4\ , \\
v^* &= 0\ .
\end{aligned}$$

3. Bose FP

$$\begin{aligned}
u^* &= 0\ , \\
v^* &= \tfrac{1}{5}(N+4)\varepsilon + \tfrac{6}{25}(N+4)\varepsilon^2 + \tfrac{1}{1250}(N+4)\Big(103 - 384\zeta(3)\Big)\varepsilon^3 \\
&+ \tfrac{1}{6250}(N+4)\Big(-3296\zeta(3) - 1920\zeta(4) + 12160\zeta(5) + 265\Big)\varepsilon^4\ .
\end{aligned}$$

4. Complex cubic FP

$$\begin{aligned}
u^* &= \tfrac{1}{5N-4}(N+4)\varepsilon + \tfrac{1}{(4-5N)^3}(N+4)\Big(70N^2 - 205N + 139\Big)\varepsilon^2 \\
&+ \tfrac{1}{4(5N-4)^5}(N+4)\Big(48\zeta(3)(5N-4)\Big(64N^3 - 188N^2 + 151N \\
&- 23\Big) - 6370N^4 - 24149N^3 + 144719N^2 - 197208N + 83256\Big)\varepsilon^3 \\
&+ \tfrac{1}{12(5N-4)^7}(N+4)\Big(48\zeta(3)\Big(22200N^6 + 110580N^5 - 754767N^4 \\
&+ 1326821N^3 - 887361N^2 + 132711N + 49508\Big) + 144\zeta(4)(5N-4)^3 \\
&\times \Big(96N^3 - 276N^2 + 221N - 37\Big) - 480\zeta(5)\Big(15200N^6 \\
&- 79920N^5 + 168488N^4 - 183439N^3 + 109827N^2 - 34792N \\
&+ 4656\Big) - 99250N^6 + 2692575N^5 - 36725295N^4 + 140337844N^3 \\
&- 230649570N^2 + 174742836N - 50310868\Big)\varepsilon^4\ ,
\end{aligned}$$

$$
\begin{aligned}
v^* &= \tfrac{1}{5N-4}(N+4)(N-2)\varepsilon + \tfrac{1}{(5N-4)^3}(N+4)\Big(30N^3+25N^2\\
&- 217N+166\Big)\varepsilon^2 + \tfrac{1}{4(4-5N)^5}(N+4)\Big(96\zeta(3)(5N-4)\Big(8N^4+16N^3\\
&- 88N^2+75N-9\Big) - 1030N^5 - 2751N^4 - 46033N^3\\
&+ 2\Big(103795N^2 - 133668N + 54904\Big)\Big)\varepsilon^3\\
&+ \tfrac{1}{12(4-5N)^7}(N+4)\Big(48\zeta(3)\Big(10300N^7 - 39580N^6 + 328467N^5\\
&- 1208826N^4 + 1806293N^3 - 1074204N^2 + 94274N + 82968\Big)\\
&+ 144\zeta(4)\Big(2000N^7 + 5200N^6 - 58660N^5 + 142451N^4\\
&- 160415N^3 + 92612N^2 - 25680N + 2496\Big) - 480\zeta(5)\Big(3800N^7\\
&- 2280N^6 - 47048N^5 + 134947N^4 - 162421N^3 + 101278N^2\\
&- 32704N + 4448\Big) - 39750N^7 + 1242425N^6 - 3090975N^5\\
&- 35240910N^4 + 171686590N^3 - 295865304N^2\\
&+ 226373292N - 65077096\Big)\varepsilon^4 \ .
\end{aligned}
$$

The stability properties of the FPs are controlled by the eigenvalue exponents ω_i of the stability matrix

$$
\Omega = \begin{pmatrix} \frac{\partial\beta_u(u,v)}{\partial u} & \frac{\partial\beta_u(u,v)}{\partial v} \\ \frac{\partial\beta_v(u,v)}{\partial u} & \frac{\partial\beta_v(u,v)}{\partial v} \end{pmatrix} \tag{13}
$$

taken at the given FPs. We calculated the eigenvalue exponents at all the FPs. For the most intriguing Bose and complex cubic FPs they are [30]

$$
\begin{aligned}
\omega_1 &= -\frac{1}{2}\varepsilon + \frac{6}{20}\varepsilon^2 + \frac{1}{8}\Big[-\frac{257}{125} - \frac{384}{125}\zeta(3)\Big]\varepsilon^3\\
&+ \frac{1}{16}\Big[\frac{5109}{1250} + \frac{624}{125}\zeta(3) - \frac{576}{125}\zeta(4) + \frac{3648}{125}\zeta(5)\Big]\varepsilon^4 \ ,\\
\omega_2 &= \frac{1}{10}\varepsilon - \frac{14}{100}\varepsilon^2 + \frac{1}{8}\Big[-\frac{311}{625} + \frac{768}{625}\zeta(3)\Big]\varepsilon^3\\
&+ \frac{1}{16}\Big[-\frac{61}{250} + \frac{3752}{3125}\zeta(3) + \frac{1152}{625}\zeta(4) - \frac{4864}{625}\zeta(5)\Big]\varepsilon^4
\end{aligned} \tag{14}
$$

at the Bose FP and

$$
\begin{aligned}
\omega_1 &= -\frac{1}{2}\varepsilon + \frac{\Big(60N^3 - 160N^2 + 181N - 85\Big)}{4(5N-4)^2(2N-1)}\varepsilon^2\\
&- \frac{1}{16(2N-1)^3(5N-4)^4}\Big[\Big(30720N^7 - 178176N^6 + 456960N^5\\
&- 648384N^4 + 534336N^3 - 251952N^2 + 62640N - 6336\Big)\zeta(3)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 20560N^7 - 165328N^6 + 644392N^5 - 1406864N^4 \\
&+ \quad 1756745N^3 - 1224341N^2 + 433704N - 59052\Big]\varepsilon^3 \\
&+ \quad \frac{1}{64(2N-1)^5(5N-4)^6}\Big[\Big(9984000N^{11} - 149237760N^{10} \\
&+ \quad 921189888N^9 - 3096268032N^8 + 6362845824N^7 \\
&- \quad 8473037952N^6 + 7511188512N^5 - 4452728592N^4 \\
&+ \quad 1734564864N^3 - 423296208N^2 + 58187520N \\
&- \quad 3401472\Big)\zeta(3) + \Big(-9216000N^{11} + 77414400N^{10} \\
&- \quad 299013120N^9 + 693626112N^8 - 1064454912N^7 \\
&+ \quad 1127834496N^6 - 838693440N^5 + 436803408N^4 \\
&- \quad 155991312N^3 + 36370368N^2 - 4983552N \\
&+ \quad 304128\Big)\zeta(4) + \Big(58368000N^{11} - 501964800N^{10} \\
&+ \quad 1976709120N^9 - 4653603840N^8 + 7224314880N^7 \\
&- \quad 7735272960N^6 + 5820821760N^5 - 3078575040N^4 \\
&+ \quad 1122437280N^3 - 268990560N^2 + 38181120N \\
&- \quad 2434560\Big)\zeta(5) + 8174400N^{11} - 111638720N^{10} \\
&+ \quad 782999600N^9 - 3263360352N^8 + 8565392076N^7 \\
&- \quad 14703626172N^6 + 16826740917N^5 - 12839720112N^4 \\
&+ \quad 6396391932N^3 - 1976965857N^2 + 340410916N \\
&- \quad 24804436\Big]\varepsilon^4 \ ,
\end{aligned}
$$

$$
\begin{aligned}
\omega_2 \quad = & \quad -\frac{(N-2)}{2(5N-4)}\varepsilon + \frac{(N-1)\Big(140N^3 - 600N^2 + 567N - 70\Big)}{4(2N-1)(5N-4)^3}\varepsilon^2 \\
&- \quad \frac{(N-1)}{16(2N-1)^3(5N-4)^5}\Big[\Big(61440N^7 - 402432N^6 + 975744N^5 \\
&- \quad 1193856N^4 + 827808N^3 - 337536N^2 + 77424N - 7872\Big)\zeta(3) \\
&- \quad 24880N^7 + 140544N^6 - 550920N^5 + 1554928N^4 \\
&- \quad 2500155N^3 + 2090850N^2 - 828628N + 118872\Big]\varepsilon^3 \\
&- \quad \frac{(N-1)}{64(2N-1)^5(5N-4)^7}\Big[\Big(12006400N^{11} - 89251840N^{10} \\
&+ \quad 476114176N^9 - 1821753600N^8 + 4343448768N^7 \\
&- \quad 6373076352N^6 + 5838538224N^5 - 3325387152N^4 \\
&+ \quad 1126329168N^3 - 198020768N^2 + 10046816N \\
&+ \quad 986752\Big)\zeta(3) + \Big(18432000N^{11} - 168652800N^{10} \\
&+ \quad 652515840N^9 - 1439023104N^8 + 2036938752N^7
\end{aligned}
$$

$$
\begin{aligned}
&- \quad 1962510336N^6 + 1324196640N^5 - 630472896N^4 \\
&+ \quad 209021904N^3 - 46238400N^2 + 6172416N \\
&- \quad 377856\Big)\zeta(4) + \Big(-77824000N^{11} + 679014400N^{10} \\
&- \quad 2474536960N^9 + 5043287040N^8 - 6405066240N^7 \\
&+ \quad 5277569280N^6 - 2800588800N^5 + 883784640N^4 \\
&- \quad 113276640N^3 - 20895040N^2 + 9518080N \\
&- \quad 1008640\Big)\zeta(5) - 2440000N^{11} - 83641600N^{10} \\
&+ \quad 1064384720N^9 - 5509282032N^8 + 16208366916N^7 \\
&- \quad 29918087352N^6 + 35918140899N^5 - 28188817515N^4 \\
&+ \quad 14172784689N^3 - 4333185934N^2 + 721222916N \\
&- \quad 49452712\Big]\varepsilon^4
\end{aligned} \tag{15}
$$

at the complex cubic one.

It is known that RG series are at best asymptotic, the coefficients of the series $\sum_k f_k g^k$ behave at large k as $k!k^b(-a)^k$ [18, 31, 32]. An appropriate resummation procedure has to be applied, in order to extract reliable physical information from them. To obtain the eigenvalues estimates, we have used an approach based on the Borel-Leroy transformation

$$
F(\varepsilon; a, b) = \sum_{k=0}^{\infty} A_k(\lambda) \int_0^\infty e^{-\frac{x}{a\varepsilon}} \Big(\frac{x}{a\varepsilon}\Big)^b d\Big(\frac{x}{a\varepsilon}\Big) \frac{z^k(x)}{[1 - z(x)]^{2\lambda}} \tag{16}
$$

modified with the conformal mapping $z = \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$ [25, 26], which does not require the knowledge of the exact asymptotic high-order behavior of the series [28]. The coefficients $A_k(\lambda)$ are determined from the equality $B(x(z)) = \frac{A(\lambda,z)}{(1-z)^{2\lambda}}$ where the Borel-Leroy transform $B(x)$ is the analytical continuation of the series $\sum_k \frac{f_k}{a^k\Gamma(b+k+1)} x^k$ absolutely convergent in the unit circle, f_k are the coefficients of the original series. The numbers a and b characterize the main divergent part of the series. Since, in practice, we deal with a piece of the series only where the asymptotic regime might not be established, we vary parameters a and b in a neighborhood of their exact values. Our principle observation is that the result of processing $F(\varepsilon; a, b)$ exhibits very weak dependence on the transformation parameters a and b varying in a wide range. Thus, we put the stability of the result of processing with respect to variation of a and b into the foundation of our approach to resummation of divergent series[8] [28]. The parameter λ is chosen from the condition of the most rapid convergence of the series (16), that is from minimizing the quantity $|1 - \frac{F_l(\varepsilon;a,b)}{F_{l-1}(\varepsilon;a,b)}|$, where l is the step of truncation and $F_l(\varepsilon; a, b)$ is the l-partial sum for $F(\varepsilon; a, b)$. If both eigenvalues are negative, the FP is infrared stable and the critical behavior of experimental systems undergoing second-order phase transitions is determined only by that stable point. For the Bose and the complex cubic FPs, our numerical results are displayed in Table I.

[8] The exact values of the asymptotic parameters a and b were calculated for the $O(n)$-symmetric models [32], for the real cubic model [27], and recently for a number of more complicated models [36]. Calculations

Table I: Eigenvalue estimates obtained for the Bose (BFP) and the complex cubic (CCFP) fixed points at $N = 2$ and $N = 3$ in the four-loop order in ε ($\varepsilon = 1$) using the Borel-Leroy transformation with a conformal mapping.

Type of FP	$N=2$		$N=3$	
	ω_1	ω_2	ω_1	ω_2
BFP	$-0.395(25)$	$0.004(5)$	$-0.395(25)$	$0.004(5)$
CCFP	$-0.392(30)$	$-0.029(20)$	$-0.400(30)$	$-0.015(6)$

It is seen that the complex cubic FP is absolutely stable in $D = 3$ ($\varepsilon = 1$), while the Bose point appears to be of the "saddle" type[9]. However ω_2's of either points are very small at the four-loop level, thus implying that these points may swap their stability in the next order of RG approximation. We can compare ω_2 at the complex cubic FP quoted in Table I with the three-loop results of Ref. [10] obtained in the framework of RG approach directly in 3D. Those estimates $\omega_2 = -0.010$ for $N = 2$ and $\omega_2 = -0.011$ for $N = 3$ are solidly consistent with ours.

In addition to the eigenvalues, we have computed the critical dimensionality of the order parameter of model (2). The critical spin dimensionality N_c for the complex cubic model is determined as that value of N, at which the complex cubic fixed point coincides with the isotropic one. Equivalently, for $N = N_c$ the second eigenvalue of the stability matrix Ω taking at the complex cubic FP vanishes, $\omega_2 = 0$. The four-loop ε expansion reads

$$N_c = 2 - \varepsilon + \frac{5}{24}\Big[6\zeta(3) - 1\Big]\varepsilon^2 + \frac{1}{144}\Big[45\zeta(3) + 135\zeta(4) - 600\zeta(5) - 1\Big]\varepsilon^3.$$

Instead of processing this expression numerically, we have established the exact relation $N_c = \frac{1}{2}n_c$ between the critical spin dimensionalities of the complex and the real hypercubic models, which is independent on the order of approximation used. In fact, both models, the complex and the real cubic ones, exhibit effectively the isotropic critical behavior at $N = N_c$ and $n = n_c$, respectively. Therefore, because the complex $O(2N)$-symmetric model is equivalent to the real $O(n)$-symmetric one, the relation $2N_c = n_c$ holds true. For $N > N_c$ the complex cubic FP of model (2) should be stable in 3D.

The five-loop ε expansion for n_c was recently obtained in Ref. [33]. Resummation of that series gave the estimate $n_c = 2.894(40)$ (see Ref. [34]). Therefore we conclude that $N_c = 1.447(20)$ from the five-loops. Practically the same estimate $N_c = 1.435(25)$ follows from a constrained analysis of n_c taking into account $n_c = 2$ in two dimensions [27]. From the recent pseudo-ε expansion analysis of the real hypercubic model [35] one can extract $N_c = 1.431(3)$. However the most accurate estimate $N_c = 1.445(20)$ results from the value $n_c = 2.89(4)$ obtained on the basis of the numerical analysis of the four-loop [34] and the

show that the resummation method employed in this paper gives the same results as the conventional technique using the exact values of the asymptotic parameters (see Refs. [27] and [28])

[9]The fixed point is called to be of the "saddle" type provided their eigenvalue exponents ω_1 and ω_2 are of opposite sings at the (u, v) plane.

six-loop [27] 3D RG expansions for the β-functions of the real hypercubic model. Since $N_c < 2$, the critical thermodynamics in the NbO_2 crystal, in the antiferromagnets $TbAu_2$, DyC_2, K_2IrCl_6, TbD_2, MnS_2, and Nd as well as in the gelical magnets Tb, Dy, and Ho should be controlled by the complex cubic fixed point with a specific set of critical exponents, in the frame of the given approximation.

III Critical Exponents and Conclusion

In the previous section we have shown that one of the four FPs is absolute stable in 3D. This is the anisotropic complex cubic FP. Now let us calculate the critical exponents. To do this, substitute the coordinates of the FPs found in Sec. II into the expressions for η and γ^{-1} [see Eqs. (10) and (12)]. For the stable complex cubic FP, this yields

$$\begin{aligned}
\eta &= \frac{\varepsilon^2}{4(5N-4)^2}(N-1)(2N-1) + \frac{\varepsilon^3}{16(5N-4)^4}(N-1)\Big[190N^3 \\
&- 535N^2 + 652N - 324\Big] - \frac{\varepsilon^4}{64(5N-4)^6}(N-1)\Big[96\zeta(3)\Big(160N^5 \\
&- 768N^4 + 1572N^3 - 1613N^2 + 777N - 132\Big) - 10570N^5 \\
&+ 22691N^4 + 68527N^3 - 280399N^2 + 326888N - 127676\Big], \qquad (17)
\end{aligned}$$

$$\begin{aligned}
\gamma^{-1} &= 1 - \frac{3\,\varepsilon}{2(5N-4)}(N-1) + \frac{\varepsilon^2}{4(5N-4)^3}(N-1)\Big[40N^2 - 214N \\
&+ 205\Big] - \frac{\varepsilon^3}{8(5N-4)^5}(N-1)\Big[12\zeta(3)(5N-4)\Big(32N^3 - 156N^2 \\
&+ 159N - 13\Big) - 940N^4 - 6748N^3 + 42681N^2 - 67102N \\
&+ 32558\Big] - \frac{\varepsilon^4}{64(5N-4)^7}(N-1)\Big[8\zeta(3)(5N-4)\Big(5650N^5 \\
&+ 58655N^4 - 303683N^3 + 396967N^2 - 96502N - 65510\Big) \\
&+ 72\zeta(4)(5N-4)^3\Big(32N^3 - 156N^2 + 159N - 13\Big) \\
&- 160\zeta(5)(5N-4)^2\Big(304N^4 - 1616N^3 + 2424N^2 - 1277N \\
&+ 265\Big) - 54300N^6 - 17220N^5 - 7422827N^4 + 42427564N^3 \\
&- 86521137N^2 + 76563448N - 25005620\Big]. \qquad (18)
\end{aligned}$$

The other critical exponents can be found through the known scaling relations. The numerical estimates obtained via transformation (16) are collected in Table II. The critical exponents for the isotropic and Bose FPs are also presented, for comparison. We can compare our results with the available experimental data. For example, in the case of the structural transition in NbO_2 crystal, the critical exponent of spontaneous polarization was

Table II: Critical exponents for the isotropic (IFP), the Bose (BFP), and the complex cubic (CCFP) FPs at $N = 2$ and $N = 3$ calculated in the four-loop order in ε ($\varepsilon = 1$) using the Borel-Leroy transformation with a conformal mapping.

Type of FP	$N = 2$			$N = 3$		
	η	ν	γ	η	ν	γ
IFP	0.0343(20)	0.725(15)	1.429(20)	0.0317(10)	0.775(15)	1.524(25)
BFP	0.0348(10)	0.664(7)	1.309(10)	0.0348(10)	0.664(7)	1.309(10)
CCFP	0.0343(20)	0.715(10)	1.404(25)	0.0345(15)	0.702(10)	1.390(25)

measured in Ref. [8], $0.33 < \beta < 0.44$. Our estimate $\beta = 0.371$ obtained using the data of Table II and scaling relations lies in that interval.

In conclusion, the four-loop ε-expansion analysis of the Ginzburg-Landau model with cubic anisotropy and complex vector order parameter relevant to the phase transitions in certain antiferromagnets with complicated ordering has been carried out. Investigation of the global structure of RG flows for the physically significant cases $N = 2$ and $N = 3$ has shown that the anisotropic complex cubic FP is absolutely stable in 3D. Therefore the critical thermodynamics of the phase transitions of concern is governed by this stable point with specific critical exponents. The critical dimensionality $N_c = 1.445(20)$ obtained from six loops supports this conclusion. At the complex cubic FP, the critical exponents were calculated using the Borel-Leroy resummation technique in combination with a conformal mapping. For the structural phase transition in NbO_2, for the antiferromagnetic phase transitions in $TbAu_2$ and DyC_2 as well as for the phase transitions in the rare-earth metals Ho, Dy, and Tb, they were shown to be close to the critical exponents of the $O(4)$-symmetric model. In contrast to this, the critical exponents for the antiferromagnetic phase transitions in K_2IrCl_6, TbD_2, MnS_2, and Nd turned out to be close to the Bose ones.

Although our calculations show that the complex cubic FP, rather than the Bose one, is stable at the four-loop level, the eigenvalues $|\omega_2|$ of both points are very small. Therefore the situation is very close to marginal, and the FPs might change their stability to opposite in the next order of perturbation theory, so that the Bose point would occur stable. This conjecture is in agreement with the recent six-loop RG study of three-coupling-constant model (1) directly in three dimensions [36]. There, the authors argue the global stability of the Bose FP, although the numerical estimate $\omega_2 = -0.007(8)$ of the smallest stability matrix eigenvalue at the Bose FP appears to be very small and the apparent accuracy of the analysis does not exclude the opposite sign for ω_2. In this situation, it would be highly desirable to compare the critical exponents values obtained theoretically with values that could be determined from experiments, in order to verdict which of the two FPs is really stable in physical space. Finally, it would be also useful to investigate certain universal amplitude ratios of the model because they vary much more among different universality classes than exponents do and might be more effective as a diagnostic tool.

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